Color Glass Condensate and Glasma

BNL, May 2010

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1. **Color Glass Condensate**
   - Why small-x gluons matter
   - Color Glass Condensate
2. **Just before the collision: Factorization**
   - Stages of AA collisions
   - Leading Order
   - Leading Log resummation
3. **Just after the collision: Glasma fields**
   - Initial color fields
   - Link to the Lund model
   - The ridge in Au-Au collisions
4. **Matching to hydrodynamics**
   - Glasma instabilities
   - Hydro in a toy model

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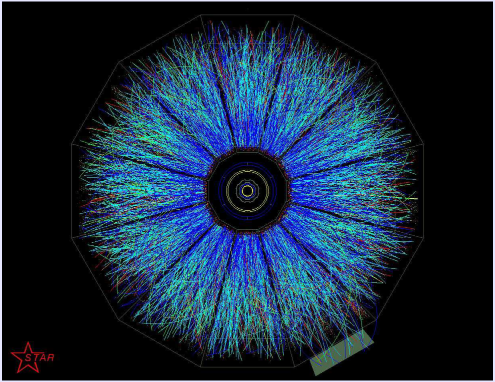
Longitudinal momentum fraction in AA collisions

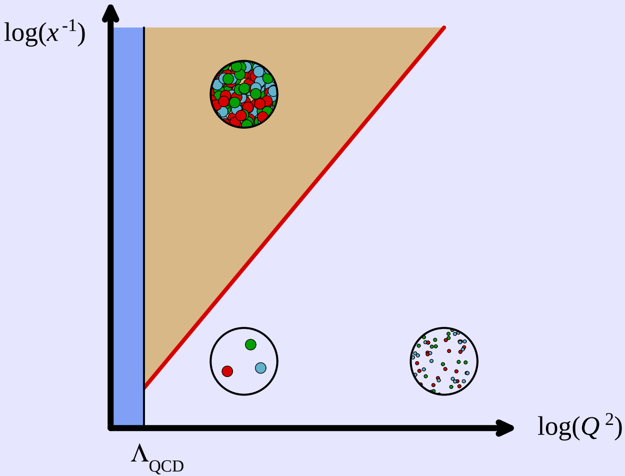
Nucleus-Nucleus collision



- 99% of the multiplicity below $p_\perp \sim 2$ GeV
- $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV)
- $x \sim 4.10^{-4}$ at the LHC ($\sqrt{s} = 5.5$ TeV)

  $\triangleright$ partons at small x are the most important

Saturation domain



Implications for a QCD approach

- Main difficulty: How to treat collisions involving a large number of partons?

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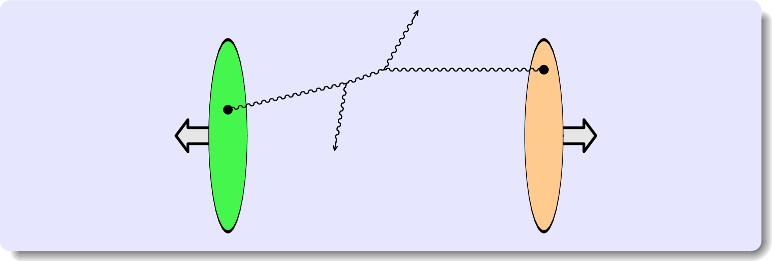
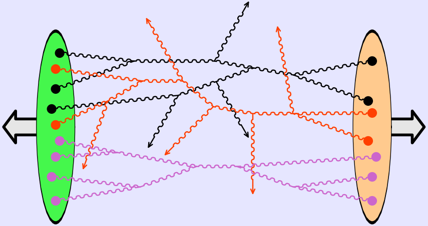
Implications for a QCD approach



- Main difficulty: How to treat collisions involving a large number of partons?

- Dilute regime : one parton in each projectile interact (what the standard perturbative techniques are made for)

Implications for a QCD approach



- Main difficulty: How to treat collisions involving a large number of partons?

- Dense regime : multiparton processes become crucial
  ▷ new techniques are required
  ▷ multi-parton distributions are needed

CGC: Degrees of freedom

CGC = effective theory of small x gluons

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu +}\rho(x^-, \vec{\boldsymbol{x}}_\perp) \qquad\qquad (0 < x^- < 1/\Lambda^+)$$

- Slow partons ($k^+ < \Lambda^+$) cannot be considered static over the time-scales of the collision process
  ▷ must be treated as standard gauge fields
  ▷ eikonal coupling to the current J^μ : $A_\mu J^\mu$

- The color sources ρ are random, and described by a distribution $W_{\Lambda^+}[\rho]$, with Λ^+ the longitudinal momentum that separates "soft" and "hard"

CGC: renormalization group evolution

Independence w.r.t $\Lambda^+ \to$ evolution equation (JIMWLK) :

$$\frac{\partial W_{\Lambda^+}}{\partial \ln(\Lambda^+)} = \mathcal{H} \; W_{\Lambda^+}$$

$$\mathcal{H} = \frac{1}{2} \int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp} \frac{\delta}{\delta \alpha(\vec{\boldsymbol{y}}_\perp)} \eta(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp) \frac{\delta}{\delta \alpha(\vec{\boldsymbol{x}}_\perp)}$$

where $\quad -\boldsymbol{\partial}_\perp^2 \, \alpha(\vec{\boldsymbol{x}}_\perp) = \rho(1/\Lambda^+, \vec{\boldsymbol{x}}_\perp)$

- $\eta(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp)$ is a non-linear functional of ρ
- Resums all the powers of $\alpha_s \ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- Simplifies into the BFKL equation when the source ρ is small (expand η in powers of ρ)

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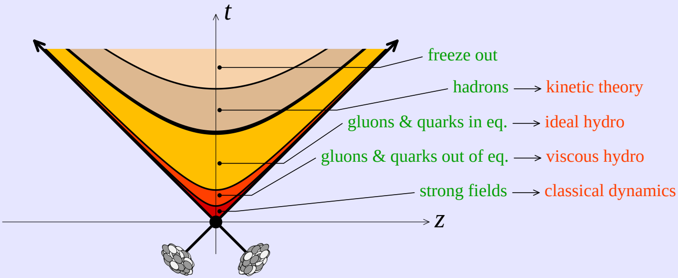
Stages of a nucleus-nucleus collision



- The Color Glass Condensate provides a framework to describe nucleus-nucleus collisions up to a time $\tau \sim Q_s^{-1}$
- Subsequent stages are described as fluid dynamics

Reminder on hydrodynamics

Equations of hydrodynamics :

$$\partial_\mu T^{\mu\nu} = 0$$

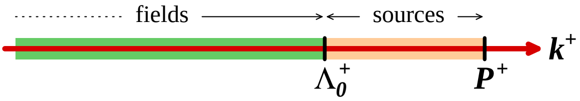
Additional inputs :

EoS : $p = f(\epsilon)$, Transport coefficients : $\eta, \zeta, \cdots$

- Required initial conditions : $T^{\mu\nu}(\tau = \tau_0, \eta, \vec{x}_\perp)$

Initial conditions from CGC: power counting

- CGC effective theory with cutoff at the scale Λ_0^+ :



$$\mathcal{L} = \underbrace{-\frac{1}{2}\,\mathrm{tr}\,F_{\mu\nu}F^{\mu\nu}}_{\text{gluon dynamics}} + \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}}\;A_\mu$$

- Expansion in g^2 in the saturated regime:

$$T^{\mu\nu} = \frac{Q_s^4}{g^2}\,\Big[c_0 + c_1\,g^2 + c_2\,g^4 + \cdots\Big]$$

Initial condition from CGC: Leading Order

- The Leading Order contribution is given by classical fields :

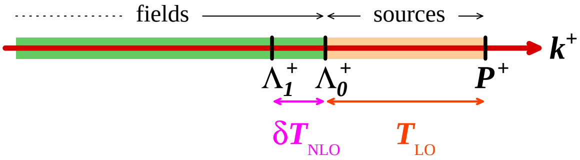
$$T^{\mu\nu}_{\rm LO} \equiv c_0 \frac{Q_s^4}{g^2} = \frac{1}{4} g^{\mu\nu}\, \mathcal{F}^{\lambda\sigma}\mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda}\mathcal{F}^{\nu}{}_{\lambda}$$

$$\text{with} \quad \underbrace{\big[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\big] = J^\nu}_{\text{Yang-Mills equation}} \quad , \quad \lim_{t\to-\infty} \mathcal{A}^\mu(t,\vec{x}) = 0$$

- The Yang-Mills equations have been solved numerically
Krasnitz, Venugopalan (1998-2000)
Lappi (2003)
Krasnitz, Nara, Venugopalan (2001-2003)

Initial condition from CGC: Leading Logs

- Consider now quantum corrections to the previous result, restricted to modes with $\Lambda_1^+ < k^+ < \Lambda_0^+$:



- At leading log accuracy, the contribution of the quantum modes in that strip can be written as :

$$\delta T^{\mu\nu}_{\rm NLO} = \left[\ln\left(\frac{\Lambda_0^+}{\Lambda_1^+}\right) \mathcal{H}_1 + \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right) \mathcal{H}_2 \right] T^{\mu\nu}_{\rm LO}$$

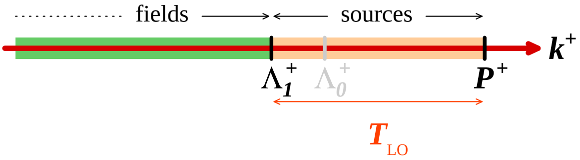
(FG, Lappi, Venugopalan (2008))

Initial condition from CGC: Leading Logs

- These corrections can be absorbed in the LO result,

$$\Big\langle \boldsymbol{T}_{\rm LO} + \delta\boldsymbol{T}_{\rm NLO} \Big\rangle_{\Lambda_0} = \Big\langle \boldsymbol{T}_{\rm LO} \Big\rangle_{\Lambda_1}$$

provided one defines a new effective theory with a lower cutoff $\Lambda_1^\pm$ and an extended distribution of sources $W_{\Lambda_1^\pm}[\rho]$:



$$W_{\Lambda_1^\pm} \equiv \left[1 + \ln\left(\frac{\Lambda_0^\pm}{\Lambda_1^\pm}\right) \mathcal{H}_{1,2}\right] W_{\Lambda_0^\pm}$$

(JIMWLK equation for a small change in the cutoff)

Initial condition from CGC: Leading Logs

- Iterate this step to integrate out all the slow field modes at leading log accuracy:

Energy-Momentum tensor at Leading Log accuracy

$$\big\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\big\rangle_{\rm LLog} = \int \big[D\rho_1\, D\rho_2\big]\; W_1\big[\rho_1\big]\; W_2\big[\rho_2\big]\; \underbrace{T^{\mu\nu}_{\rm LO}(\tau,\vec{\boldsymbol{x}}_\perp)}_{\text{for fixed } \rho_{1,2}}$$

- At leading log accuracy, the rapidity dependence comes entirely from the wavefunctions of the projectiles
- This factorization establishes a link to other reactions (such as DIS on a nuclear target) in the saturated regime
- Works for all sufficiently inclusive observables

Correlations in η and $\vec{x}_\perp$ at Leading Log

- The factorization valid for $\left<T^{\mu\nu}\right>$ can be extended to multi-point correlations :

$$\left<T^{\mu_1\nu_1}(\tau,\eta_1,\vec{x}_{1\perp})\cdots T^{\mu_n\nu_n}(\tau,\eta_n,\vec{x}_{n\perp})\right>_{\rm LLog} =$$
$$= \int \big[D\rho_1\, D\rho_2\big]\; W_1\big[\rho_1\big]\; W_2\big[\rho_2\big]$$
$$\times\; T^{\mu_1\nu_1}_{\rm LO}(\tau,\vec{x}_{1\perp})\cdots T^{\mu_n\nu_n}_{\rm LO}(\tau,\vec{x}_{n\perp})$$

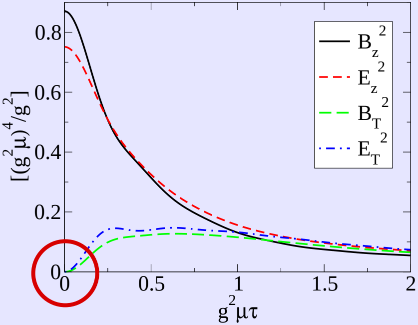
- Note: at Leading Log accuracy, all the rapidity correlations come from the evolution of the distributions $W[\rho_{1,2}]$
  $\triangleright$ they are a property of the pre-collision initial state

- This formula predicts long range ($\Delta\eta \sim \alpha_s^{-1}$) rapidity correlations for points located at the same impact parameter

Initial classical fields, Glasma

Lappi, McLerran (2006)

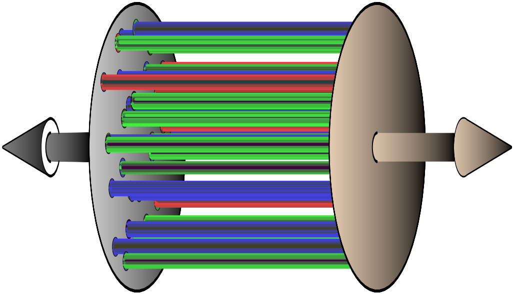
- Immediately after the collision, the chromo-$\vec{E}$ and $\vec{B}$ fields are purely longitudinal and boost invariant :



- Glasma = intermediate stage between the CGC and the quark-gluon plasma

Glasma flux tubes

- The initial chromo-$\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ fields form longitudinal "flux tubes" extending between the projectiles:



- Correlation length in the transverse plane: $\Delta r_\perp \sim Q_s^{-1}$
- Correlation length in rapidity: $\Delta\eta \sim \alpha_s^{-1}$
- The flux tubes fill up the entire volume

Glasma flux tubes

- A classical field configuration where $\boldsymbol{B}_a^i = \lambda\,\boldsymbol{E}_a^i$ has an energy-momentum tensor of the form:

$$\big\langle T^{\mu\nu}(0^+,\eta,\vec{\boldsymbol{x}})\big\rangle = \begin{pmatrix} \epsilon & & & \\ & \epsilon & & \\ & & \epsilon & \\ & & & -\epsilon \end{pmatrix}$$

- Multiplicity distribution: Negative Binomial (T. Lappi's talk)
- Long range correlations in rapidity survive in the final state
- $\boldsymbol{E}$ parallel to $\boldsymbol{B}$ $\triangleright$ non-zero $\widetilde{F}F$

Link to the Lund string model

- Tanji (2008), Fukushima, FG, Lappi (2009): the yield from the Schwinger mechanism has exactly the same form as the NLO correction in the CGC

- Analogies between the Glasma and the Lund strings:

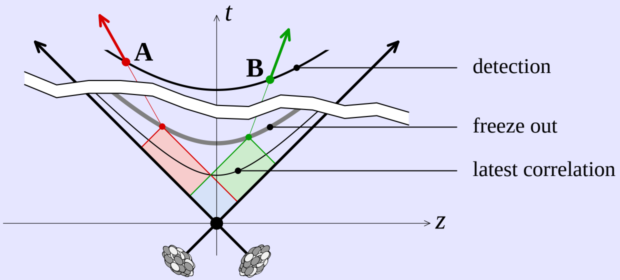
Glasma tubes	$\longleftrightarrow$	strings
Negative P_z	$\longleftrightarrow$	string tension
Glasma instability	$\longleftrightarrow$	string breaking

- Differences:
  - ***B*** field in the Glasma
  - The "string size" is set dynamically in the Glasma
  - The Glasma is more closely related to QCD

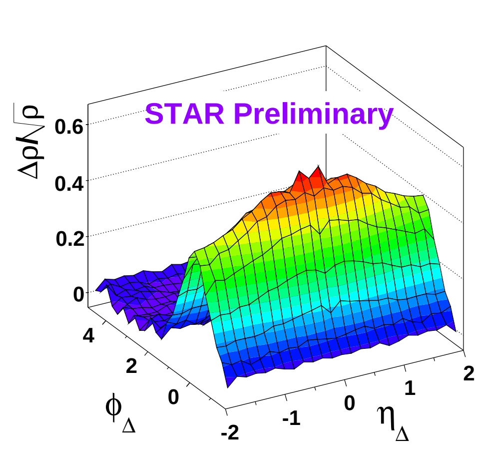
Importance of initial rapidity correlations

Early physics can survive in long range rapidity correlations

$$t_{\text{correlation}} \leq t_{\text{freeze out}} \; e^{-\frac{1}{2}|\eta_A - \eta_B|}$$



2-hadron correlations at RHIC



- Long range correlation in $\Delta\eta$ (rapidity)
- Narrow correlation in $\Delta\varphi$ (azimuthal angle)

2-hadron correlations at RHIC

Dumitru, FG, McLerran, Venugopalan (2008)
Dusling, Fernandez-Fraile, Venugopalan (2009)
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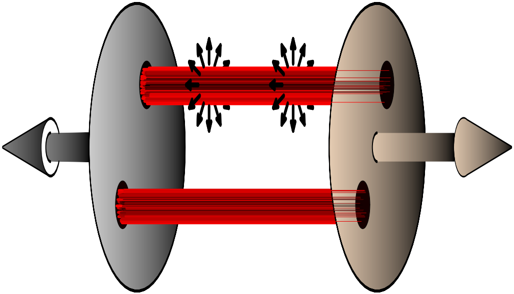
- η-independent fields lead to long range correlations :



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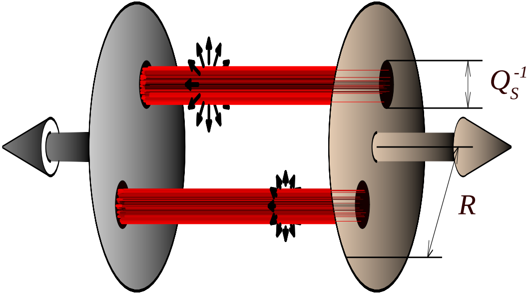

- Particles emitted by different flux tubes are not correlated
  ▷ $(RQ_s)^{-2}$ sets the strength of the correlation

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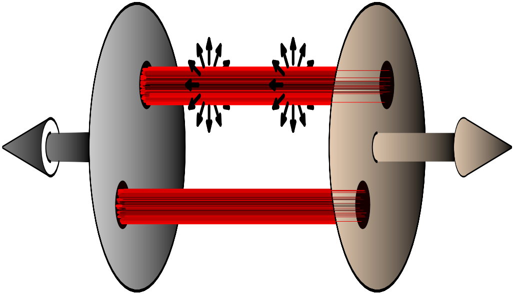

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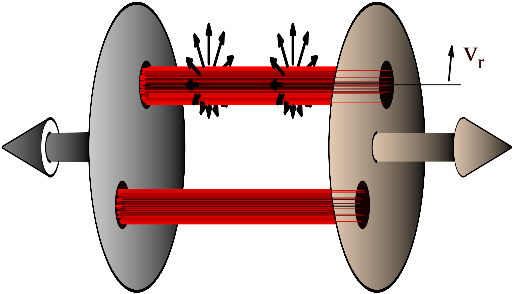


- Particles emitted by different flux tubes are not correlated
  ▷ $(RQ_s)^{-2}$ sets the strength of the correlation
- At early times, the correlation is flat in $\Delta\varphi$
  A collimation in $\Delta\varphi$ is produced later by radial flow

$T^{\mu\nu}$ of the Glasma

- Glasma:

$$\left\langle T^{\mu\nu}(0^+,\eta,\vec{\boldsymbol{x}})\right\rangle = \begin{pmatrix} \epsilon & & & \\ & \epsilon & & \\ & & \epsilon & \\ & & & -\epsilon \end{pmatrix}$$

- Ideal hydro:

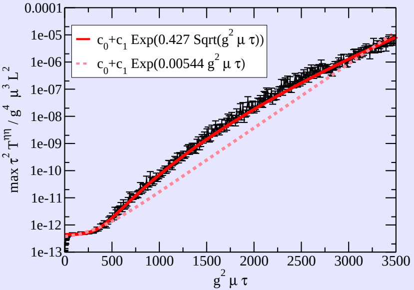
$$T^{\mu\nu}_{\rm ideal}(0^+,\eta,\vec{\boldsymbol{x}}) = \begin{pmatrix} \epsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

- If a smooth matching from the Glasma to Hydro is possible, one should be able to recover the fluid behavior from classical fields

Unstable fluctuations

Romatschke, Venugopalan (2005)

- Perturbations to the classical fields grow like $\exp(\sqrt{Q_s\tau})$ until the non-linearities become important :



▷ Quantum fluctuations become $\mathcal{O}(1)$ corrections when

$$\tau \sim \tau_{\rm max} \sim Q_s^{-1} \ln^2(1/\alpha_s)$$

Resummation of the unstable terms

- To go beyond the time $\tau_{\rm max}$, one must resum all the fastest growing terms $\sim [g^2 e^{\sqrt{Q_s\tau}}]^n$

- This amounts to superimposing fluctuations to the initial classical field:

$$
\big\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\big\rangle \underset{\substack{\rm LLog\\ \rm resummed}}{=} \int \big[D\rho_1\, D\rho_2\big]\; W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2]
\times \int \big[Da\big]\; F[a]\;\; T^{\mu\nu}_{\rm LO}[\underbrace{\mathcal{A}+a}_{\text{initial field}}]
$$

- Fukushima, FG, McLerran (2006): this result can be obtained in a semi-classical approach (with Gaussian $F[a]$)
- FG, Lappi, Venugopalan (2008): can be obtained by a resummation of the NLO result in the CGC

Hydro behavior in a toy model

- Including the fluctuations in the calculation of $T^{\mu\nu}$ is hard:
  - Space-dependent fluctuations need to be renormalized (because of UV divergences)
  - The QCD classical equations are difficult to solve

Toy model: scalar field, uniform in η

Equation of motion: $\ddot{\phi} + \frac{1}{\tau}\dot{\phi} - \boldsymbol{\nabla}_\perp^2 \phi + V'(\phi) = 0$

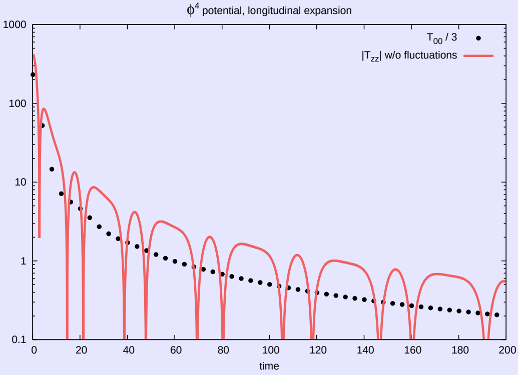
Interaction potential: $V(\phi) \sim g^2 \phi^4$

Initial conditions at $\tau = \tau_0$: $\phi = \varphi_0$, $\dot{\phi} = \dot{\varphi}_0$

Gaussian fluctuations of φ_0 and $\dot{\varphi}_0$

Hydro behavior in a toy model (1+0 dim)

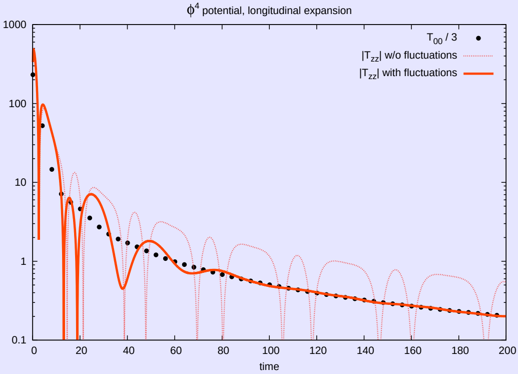
Classical field evolution w/o fluctuations



- Without fluctuations, *p* oscillates forever

Hydro behavior in a toy model (1+0 dim)

Classical field evolution with fluctuations



- Without fluctuations, *p* oscillates forever
- With fluctuations, *p* relaxes quickly to $\epsilon/3$

Hydro behavior in a toy model (1+0 dim)

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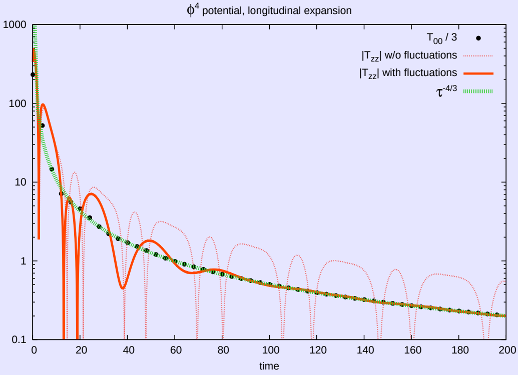
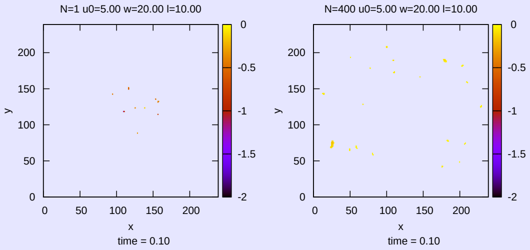


- Without fluctuations, *p* oscillates forever
- With fluctuations, *p* relaxes quickly to $\epsilon/3$
- ϵ and *p* decrease as $1/\tau^{4/3}$
  - ▷ same behavior as in ideal hydro with EoS $\epsilon = 3p$...

Hydro behavior in a toy model (1+2 dim)

- Left: $\log|(\epsilon - 3p)/\epsilon|$ without fluctuations
- Right: $\log|(\epsilon - 3p)/\epsilon|$ with fluctuations

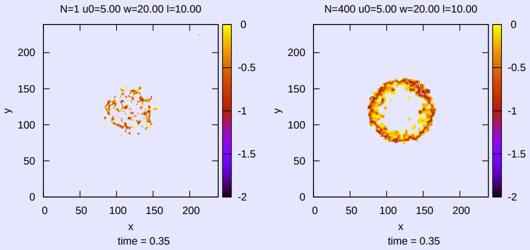
time = 0.1



Hydro behavior in a toy model (1+2 dim)

- Left: $\log|(\epsilon - 3p)/\epsilon|$ without fluctuations
- Right: $\log|(\epsilon - 3p)/\epsilon|$ with fluctuations

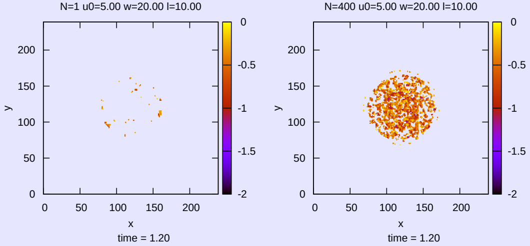
time = 0.35



Hydro behavior in a toy model (1+2 dim)

- Left: $\log|(\epsilon - 3p)/\epsilon|$ without fluctuations
- Right: $\log|(\epsilon - 3p)/\epsilon|$ with fluctuations

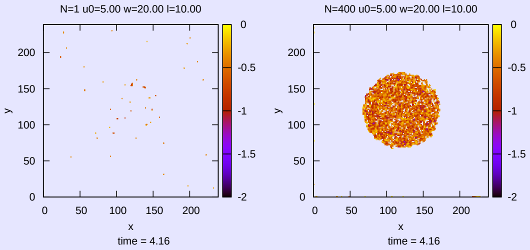
time = 1.20



Hydro behavior in a toy model (1+2 dim)

- Left: $\log|(\epsilon - 3p)/\epsilon|$ without fluctuations
- Right: $\log|(\epsilon - 3p)/\epsilon|$ with fluctuations

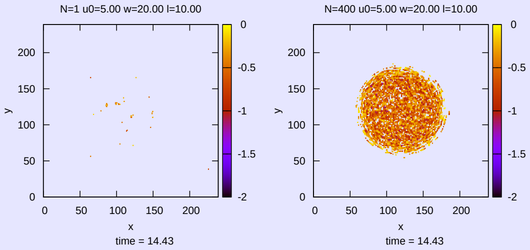
time = 4.16



Hydro behavior in a toy model (1+2 dim)

- Left: $\log|(\epsilon - 3p)/\epsilon|$ without fluctuations
- Right: $\log|(\epsilon - 3p)/\epsilon|$ with fluctuations

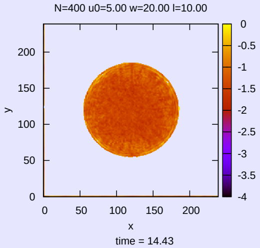
time = 14.43

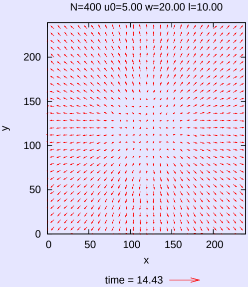


Hydro behavior in a toy model (1+2 dim)

- Left: magnitude of the viscous tensor $\log(\Pi^{\mu\nu}/\epsilon)$
- Right: velocity field

time = 14.43





Summary

Initial state, up to $\tau = 0^+$

- Consistent framework to include the non-linear saturation effects in heavy ion collisions
- Factorization of the large logs of $x_{1,2}$ into universal distributions $W[\rho]$
- Implies long range rapidity correlations, in good agreement with RHIC data

Final state evolution

- Unstable fluctuations need to be resummed, but the machinery for doing that is not fully developed
- An equation of state may be obtained by superimposing quantum fluctuations to the classical fields, without complete thermalization of the system

2-field model

$$\mathcal{L} \equiv \frac{1}{2}\left[\dot\phi_1^2 + \dot\phi_2^2\right] - \frac{g^2}{4!}\left[\phi_1^2 + \phi_2^2\right]^2$$